**Matrices – Formula Sheet:**

|  |  |
| --- | --- |
| **Systems of Equations – 2 Variables:**$$a\_{1}x+b\_{1}y=c\_{1}$$$$a\_{2}x+b\_{2}y=c\_{2}$$$$x=\frac{c\_{1}b\_{2}-b\_{1}c\_{2}}{a\_{1}b\_{2}-b\_{1}a\_{2}}$$$$y=\frac{a\_{1}c\_{2}-c\_{1}a\_{2}}{a\_{1}b\_{2}-b\_{1}a\_{2}}$$ | **Cramer’s Rule – 2x2 Matrices with Determinants:**$$D=\left|\begin{matrix}a\_{1}&b\_{1}\\a\_{2}&b\_{2}\end{matrix}\right|=a\_{1}b\_{2}-b\_{1}a\_{2}$$$$D\_{x}=\left|\begin{matrix}c\_{1}&b\_{1}\\c\_{2}&b\_{2}\end{matrix}\right|=c\_{1}b\_{2}-b\_{1}c\_{2}$$$$D\_{y}=\left|\begin{matrix}a\_{1}&c\_{1}\\a\_{2}&c\_{2}\end{matrix}\right|=a\_{1}c\_{2}-c\_{1}a\_{2}$$$$x=\frac{D\_{x}}{D} y=\frac{D\_{y}}{D}$$ |
| **Systems of Equations – 3 Variables:**$$a\_{1}x+b\_{1}y+c\_{1}z=d\_{1}$$$$a\_{2}x+b\_{2}y+c\_{2}z=d\_{2}$$$$a\_{3}x+b\_{2}y+c\_{3}z=d\_{3}$$ | $$x=\frac{d\_{1}\left(b\_{2}c\_{3}-c\_{2}b\_{3}\right)-b\_{1}\left(d\_{2}c\_{3}-c\_{2}d\_{3}\right)+c\_{1}(d\_{2}b\_{3}-b\_{2}d\_{3})}{a\_{1}\left(b\_{2}c\_{3}-c\_{2}b\_{3}\right)-b\_{1}\left(a\_{2}c\_{3}-c\_{2}a\_{3}\right)+c\_{1}(a\_{2}b\_{3}-b\_{2}a\_{3})}$$$$y=\frac{a\_{1}\left(d\_{2}c\_{3}-c\_{2}d\_{3}\right)-d\_{1}\left(a\_{2}c\_{3}-c\_{2}a\_{3}\right)+c\_{1}(a\_{2}d\_{3}-d\_{2}a\_{3})}{a\_{1}\left(b\_{2}c\_{3}-c\_{2}b\_{3}\right)-b\_{1}\left(a\_{2}c\_{3}-c\_{2}a\_{3}\right)+c\_{1}(a\_{2}b\_{3}-b\_{2}a\_{3})}$$$$z=\frac{a\_{1}\left(b\_{2}d\_{3}-d\_{2}b\_{3}\right)-b\_{1}\left(a\_{2}d\_{3}-d\_{2}a\_{3}\right)+d\_{1}(a\_{2}b\_{3}-b\_{2}a\_{3})}{a\_{1}\left(b\_{2}c\_{3}-c\_{2}b\_{3}\right)-b\_{1}\left(a\_{2}c\_{3}-c\_{2}a\_{3}\right)+c\_{1}(a\_{2}b\_{3}-b\_{2}a\_{3})}$$ |
| **Cramer’s Rule – 3x3 Matrices with Determinants:**$$D=\left|\begin{matrix}a\_{1}&b\_{1}&c\_{1}\\a\_{2}&b\_{2}&c\_{2}\\a\_{3}&b\_{3}&c\_{3}\end{matrix}\right|=a\_{1}\left|\begin{matrix}b\_{2}&c\_{2}\\b\_{3}&c\_{3}\end{matrix}\right|-b\_{1}\left|\begin{matrix}a\_{2}&c\_{2}\\a\_{3}&c\_{3}\end{matrix}\right|+c\_{1}\left|\begin{matrix}a\_{2}&b\_{2}\\a\_{3}&b\_{3}\end{matrix}\right|$$$$D=a\_{1}\left(b\_{2}c\_{3}-c\_{2}b\_{3}\right)-b\_{1}\left(a\_{2}c\_{3}-c\_{2}a\_{3}\right)+c\_{1}(a\_{2}b\_{3}-b\_{2}a\_{3})$$$$D\_{x}=\left|\begin{matrix}d\_{1}&b\_{1}&c\_{1}\\d\_{2}&b\_{2}&c\_{2}\\d\_{3}&b\_{3}&c\_{3}\end{matrix}\right|=d\_{1}\left(b\_{2}c\_{3}-c\_{2}b\_{3}\right)-b\_{1}\left(d\_{2}c\_{3}-c\_{2}d\_{3}\right)+c\_{1}(d\_{2}b\_{3}-b\_{2}d\_{3})$$$$D\_{y}=\left|\begin{matrix}a\_{1}&d\_{1}&c\_{1}\\a\_{2}&d\_{2}&c\_{2}\\a\_{3}&d\_{3}&c\_{3}\end{matrix}\right|=a\_{1}\left(d\_{2}c\_{3}-c\_{2}d\_{3}\right)-d\_{1}\left(a\_{2}c\_{3}-c\_{2}a\_{3}\right)+c\_{1}(a\_{2}d\_{3}-d\_{2}a\_{3})$$$$D\_{z}=\left|\begin{matrix}a\_{1}&b\_{1}&d\_{1}\\a\_{2}&b\_{2}&d\_{2}\\a\_{3}&b\_{3}&d\_{3}\end{matrix}\right|=a\_{1}\left(b\_{2}d\_{3}-d\_{2}b\_{3}\right)-b\_{1}\left(a\_{2}d\_{3}-d\_{2}a\_{3}\right)+d\_{1}(a\_{2}b\_{3}-b\_{2}a\_{3})$$$$x=\frac{D\_{x}}{D} y=\frac{D\_{y}}{D} z=\frac{D\_{z}}{D} D\ne 0$$ |

|  |  |
| --- | --- |
| **Matrix Addition:**$$A+B = \left|\begin{matrix}a&b\\c&d\end{matrix}\right| + \left[\begin{matrix}e&f\\g&h\end{matrix}\right] = \left|\begin{matrix}a+e&b+f\\c+g&d+h\end{matrix}\right|$$ | **Matrix Subtraction:**$$A-B = \left|\begin{matrix}a&b\\c&d\end{matrix}\right|- \left[\begin{matrix}e&f\\g&h\end{matrix}\right] = \left|\begin{matrix}a-e&b-f\\c-g&d-h\end{matrix}\right|$$ |
| **Scalar Multiplication:**$$nA=n \left|\begin{matrix}a&b\\c&d\end{matrix}\right|= \left|\begin{matrix}na&nb\\nc&nd\end{matrix}\right|$$ | **Matrix Multiplication:**$$AB = \left|\begin{matrix}a&b\\c&d\end{matrix}\right| \left[\begin{matrix}e&f\\g&h\end{matrix}\right]= \left|\begin{matrix}ae+bg&af+bh\\ce+dg&cf+dh\end{matrix}\right|$$$$2 Rows x 2 Columns=2x2 Matrix$$ |
| **MM:** $2 Rows x 2 Columns=2x2 Matrix$$$AB=\left|\begin{matrix}a&b&c\\d&e&f\end{matrix}\right| \left|\begin{matrix}g&h\\i&j\\k&l\end{matrix}\right|$$$$AB= \left|\begin{matrix}ag+bi+ck&ah+bj+cl\\dg+ei+fk&dh+ej+fl\end{matrix}\right|$$ | **MM:** $1 Row x 1 Column=1x1 Matrix$$$AB= \left|\begin{matrix}a&b&c\end{matrix}\right| \left|\begin{matrix}d\\e\\f\end{matrix}\right|$$$$AB= \left|ad+be+cf\right|$$ |
| **MM:** $3 Rows x 3 Columns=3x3 Matrix$$$AB=\left|\begin{matrix}a\\b\\c\end{matrix}\right| \left|\begin{matrix}d&e&f\end{matrix}\right|$$$$AB= \left|\begin{matrix}ad&ae&af\\bd&be&bf\\cd&ce&cf\end{matrix}\right|$$ | **MM:** $2 Rows x 4 Columns=2x4 Matrix$$$AB= \left|\begin{matrix}a&b\\c&d\end{matrix}\right| \left|\begin{matrix}e&f\\i&j\end{matrix} \begin{matrix}g&h\\k&l\end{matrix}\right|$$$$AB= \left|\begin{matrix}ae+bi&af+bj\\ce+di&cf+dj\end{matrix} \begin{matrix}ag+bk&ah+bl\\cg+dk&ch+dl\end{matrix}\right|$$ |
| **MM:** $3 Rows x 3 Columns=3x3 Matrix$$$AB=\left|\begin{matrix}a&b\\c&d\\e&f\end{matrix}\right| \left|\begin{matrix}g&h&i\\j&k&l\end{matrix}\right|$$$$AB= \left|\begin{matrix}ag+bj&ah+bk&ai+bl\\cg+dj&ch+dk&ci+dl\\eg+fj&eh+fk&ei+fl\end{matrix}\right|$$ | **MM:** $3 Rows x 3 Columns=3x3 Matrix$$$AB= \left|\begin{matrix}a&b&c\\d&e&f\\g&h&i\end{matrix}\right| \left|\begin{matrix}j&k&l\\m&n&o\\p&q&r\end{matrix}\right|$$$$AB= \left|\begin{matrix}aj+bm+cp&ak+bn+cq&al+bo+cr\\dj+em+fp&dk+en+fq&dl+eo+fr\\gj+hm+ip&gk+hn+iq&gl+ho+ir\end{matrix}\right|$$ |

|  |  |
| --- | --- |
| **Multiplicative Identity Matrices:**$$I\_{2}=\left|\begin{matrix}1&0\\0&1\end{matrix}\right|$$$$I\_{3}= \left|\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}\right|$$$$AA^{-1}=I\_{n} A^{-1}A=I\_{n}$$ | **Multiplicative Inverse of a 2x2 Matrix:**$$A=\left|\begin{matrix}a&b\\c&d\end{matrix}\right|$$$$A^{-1}=\frac{1}{ad-bc} \left|\begin{matrix}d&-b\\-c&a\end{matrix}\right|$$**Note:** $$If ad-bc=0, then A does not have a multiplicative inverse. $$ |
| **Multiplicative Inverse of a 3x3 Matrix:**$$A=\left|\begin{matrix}a&b&c\\d&e&f\\g&h&i\end{matrix}\right|$$$$A^{-1}=\frac{1}{\left|A\right|}\left|\begin{matrix}\left|\begin{matrix}e&f\\h&i\end{matrix}\right|&\left|\begin{matrix}c&b\\i&h\end{matrix}\right|&\left|\begin{matrix}b&c\\e&f\end{matrix}\right|\\\left|\begin{matrix}f&d\\i&g\end{matrix}\right|&\left|\begin{matrix}a&c\\g&i\end{matrix}\right|&\left|\begin{matrix}c&a\\f&d\end{matrix}\right|\\\left|\begin{matrix}d&e\\g&h\end{matrix}\right|&\left|\begin{matrix}b&a\\h&g\end{matrix}\right|&\left|\begin{matrix}a&b\\d&e\end{matrix}\right|\end{matrix}\right|$$$$A^{-1}=\frac{1}{\left|A\right|}\left|\begin{matrix}ei-fh&ch-bi&bf-ce\\fg-di&ai-cg&cd-af\\dh-eg&bg-ah&ae-bd\end{matrix}\right|$$ |